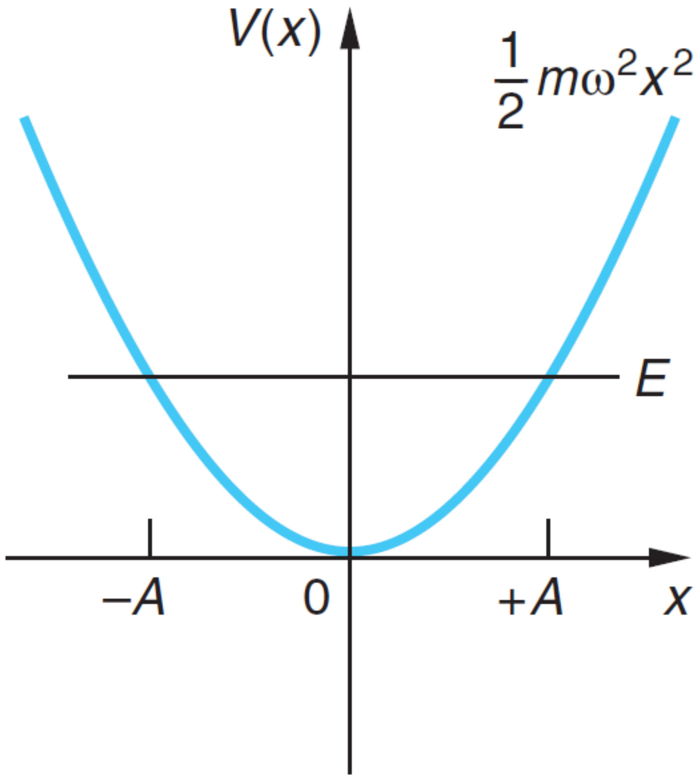
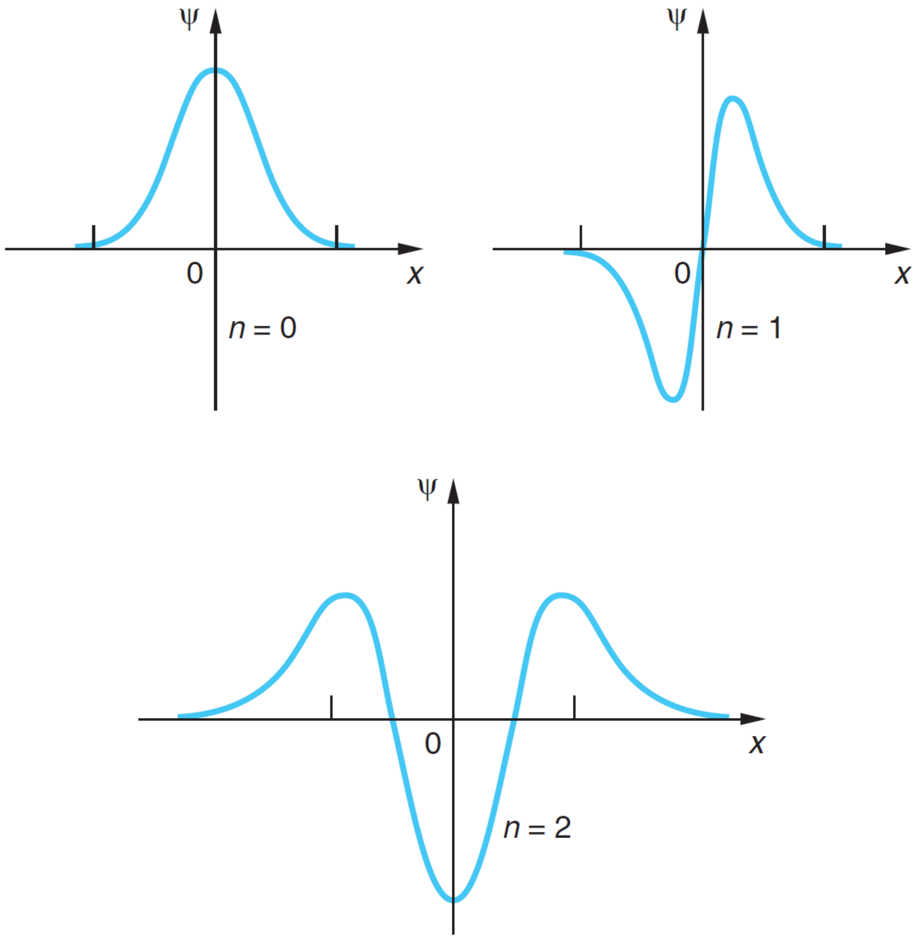
Harmonic Oscillator





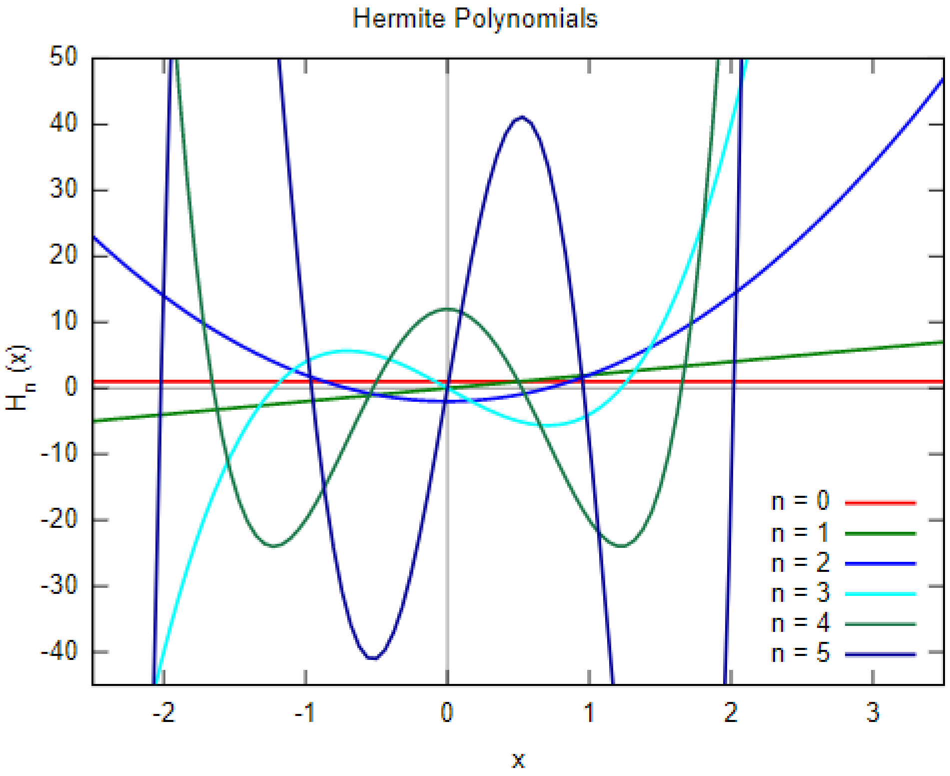
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\psi(x) = E\psi(x)$$

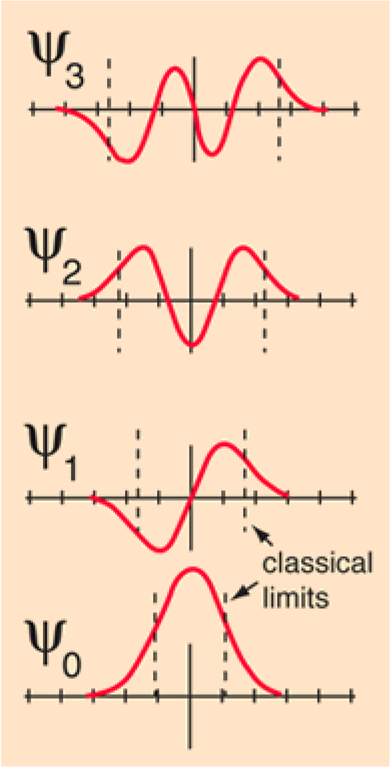
Quantum Harmonic Oscillator

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n\, n!}} H_n(\xi)\, e^{-\xi^2/2} \qquad \xi = x\sqrt{\frac{m\omega}{\hbar}}$$

Hermite Polynomials

$$H_0 = 1$$
$$H_1 = 2x$$
$$H_2 = -2 + 4x^2$$
$$H_3 = -12x + 8x^3$$
$$H_4 = 12 - 48x^2 + 16x^4$$
$$H_5 = 120x - 160x^3 + 32x^5$$
$$H_6 = -120 + 720x^2 - 480x^4 + 64x^6$$
$$H_7 = -1680x + 3360x^3 - 1344x^5 + 128x^7$$



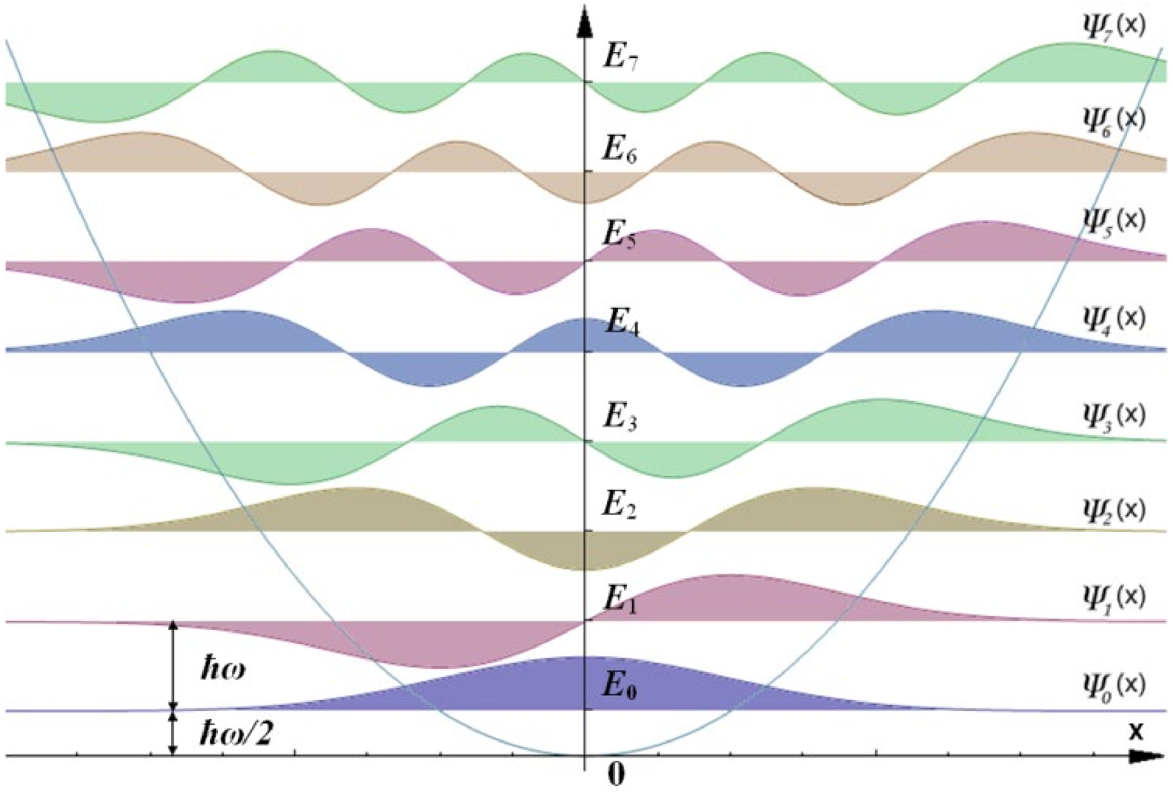


https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Map%3A_Physical_Chemistry_(McQuarrie_and_Simon)/05%3A_The_Harmonic_Oscillator_and_the_Rigid_Rotor/5.06%3A_The_Harmonic-Oscillator_Wavefunctions_involve_Hermite_Polynomials

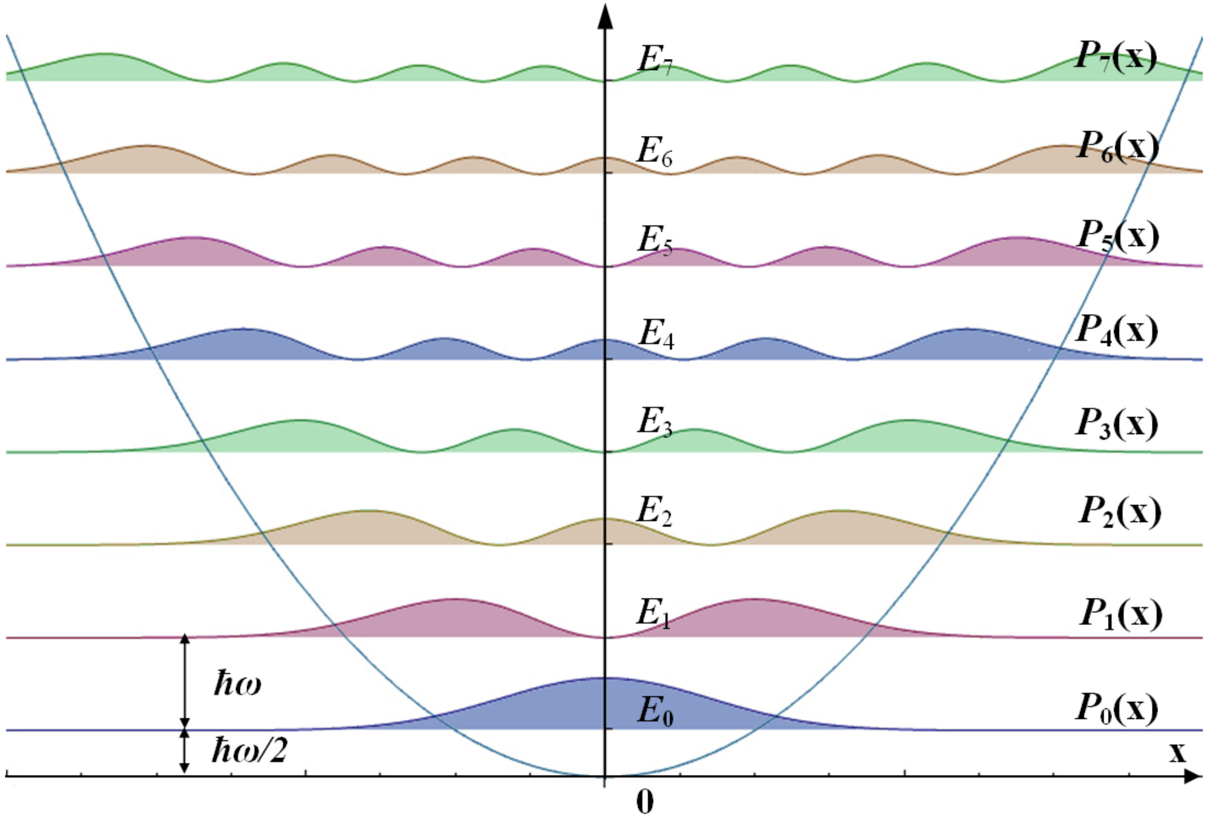
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hosc5.html

Quantum Harmonic Oscillator

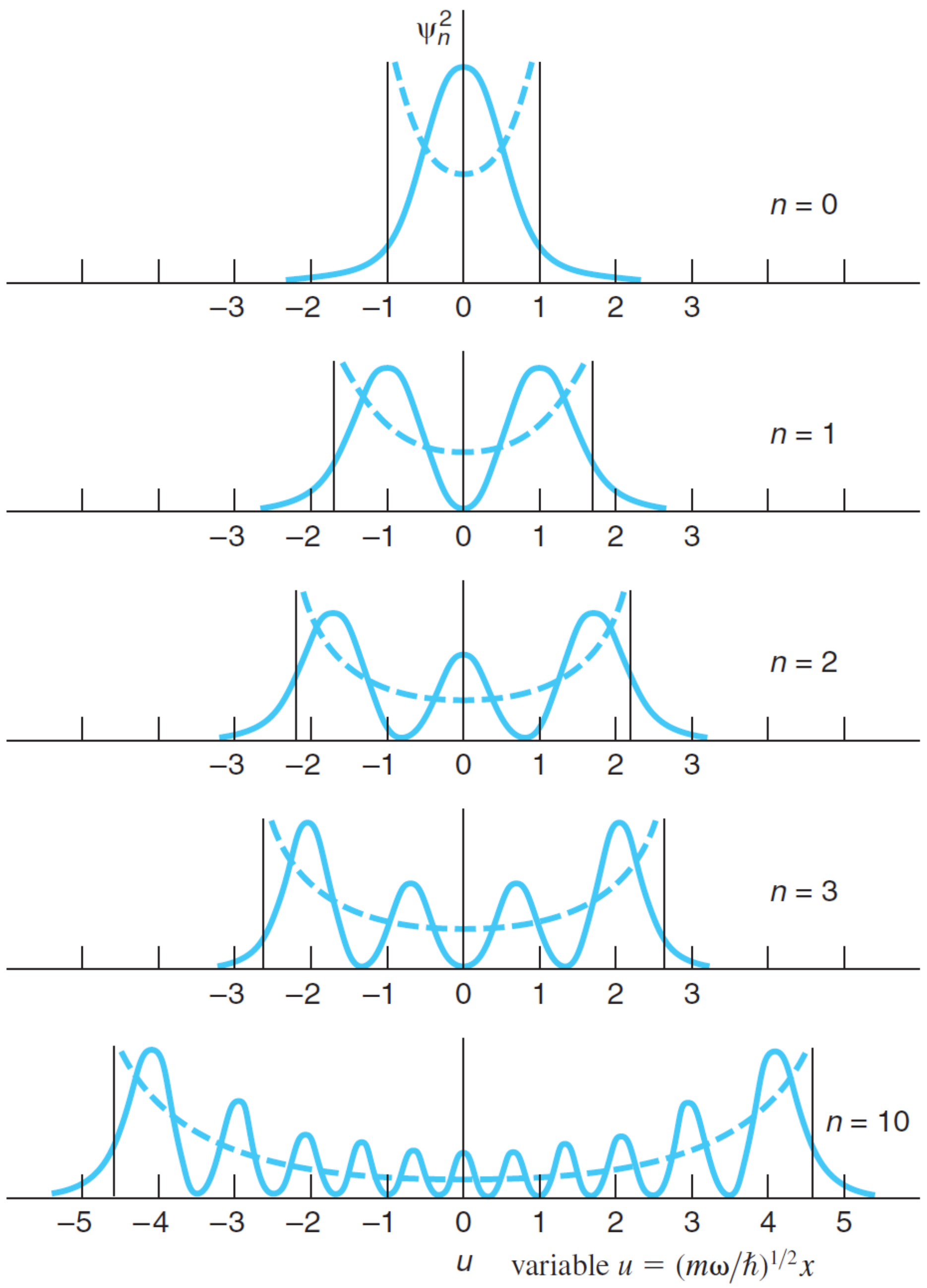
Wave Functions



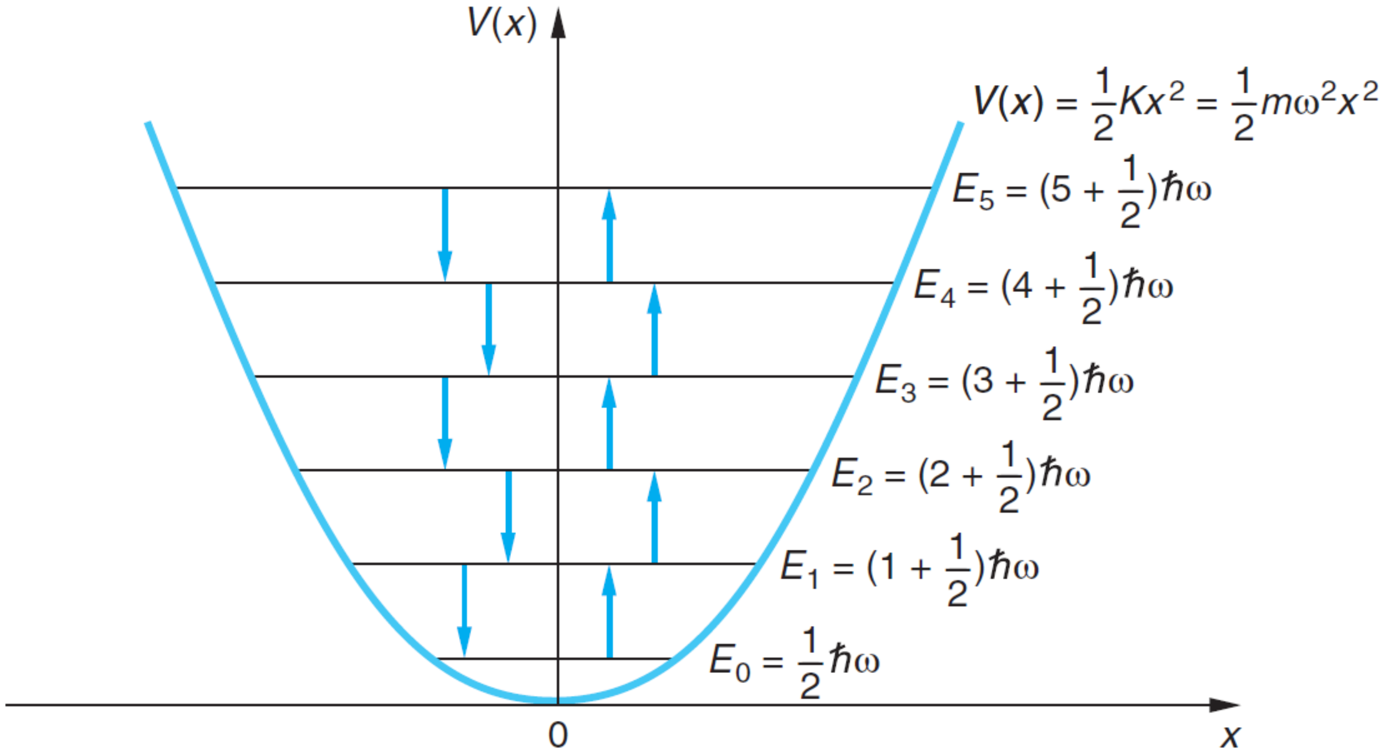
Probability Density



https://en.wikipedia.org/wiki/Quantum_harmonic_oscillator



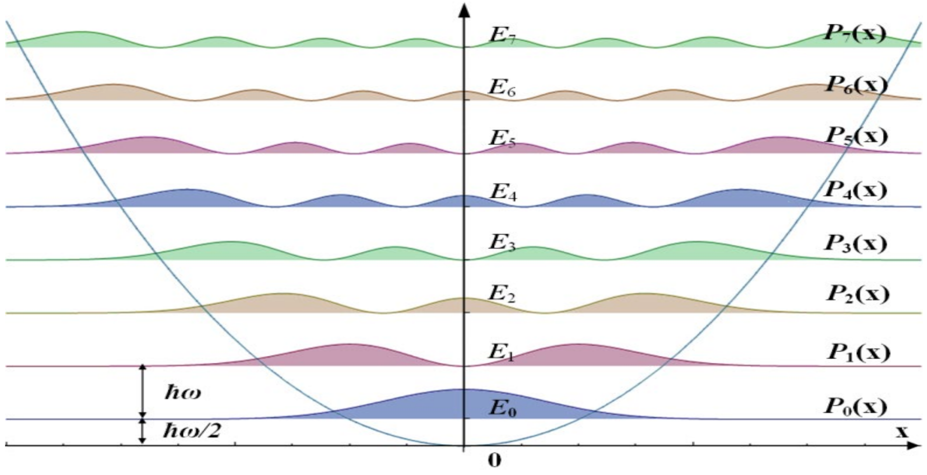
ψ_n^2
$n = 0$
$n = 1$
$n = 2$
$n = 3$
$n = 10$
–5 –4 –3 –2 –1 0 1 2 3 4 5
u variable $u = (m\omega/\hbar)^{1/2}x$

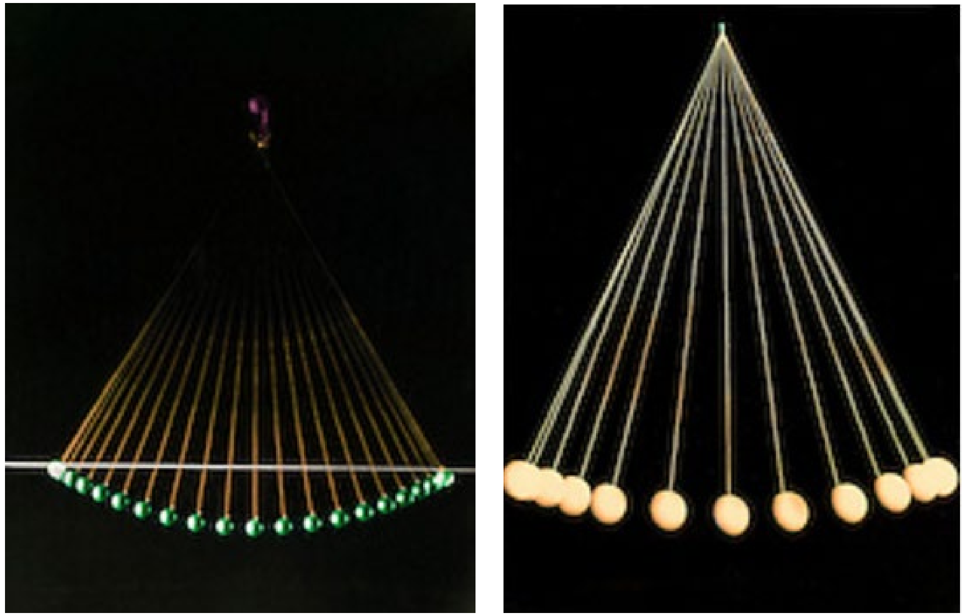


$V(x)$
$V(x) = \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2x^2$
$E_5 = (5 + \frac{1}{2})\hbar\omega$
$E_4 = (4 + \frac{1}{2})\hbar\omega$
$E_3 = (3 + \frac{1}{2})\hbar\omega$
$E_2 = (2 + \frac{1}{2})\hbar\omega$
$E_1 = (1 + \frac{1}{2})\hbar\omega$
$E_0 = \frac{1}{2}\hbar\omega$
0
x

Harmonic Oscillator Probability Densities

$|\psi_n(x)|^2$

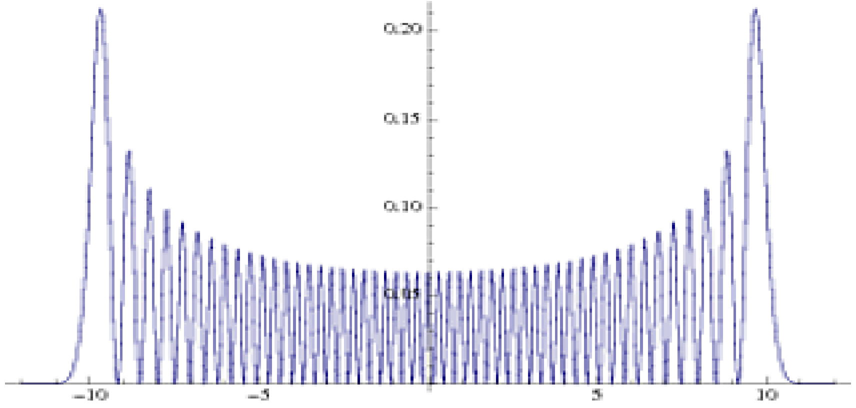




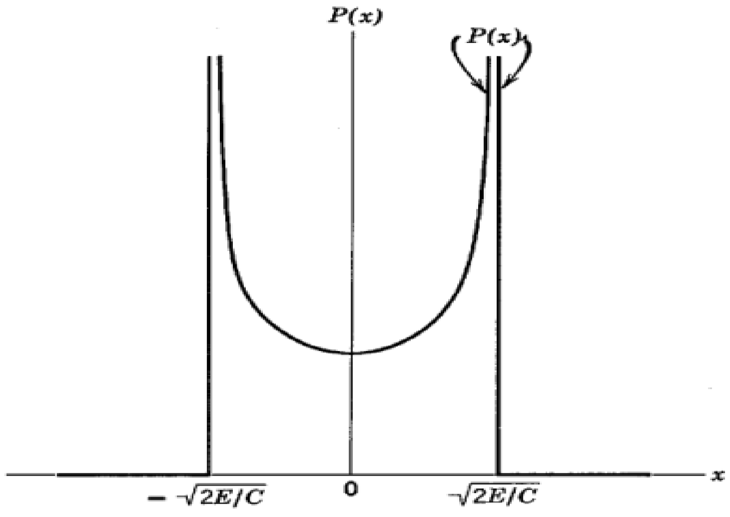
http://fphoto.photoshelter.com/gallery/Harmonic-Motion/G0000ThspLLmT3rc/C0000lwydqJrTnzw

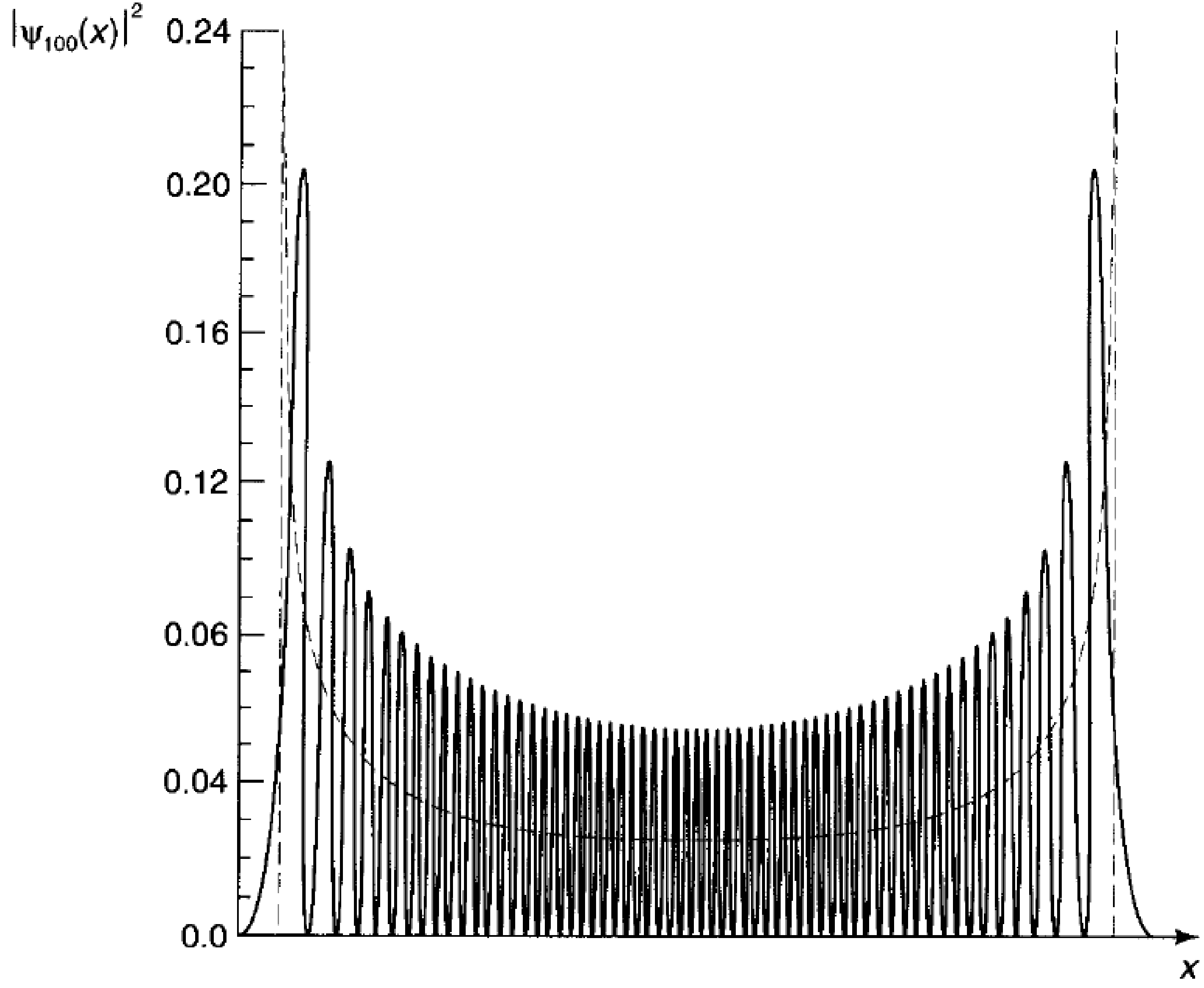
Classical harmonic
Oscillator probability P(x)

$|\psi_{50}(x)|^2$



x





(b)

Figure 2.5: (a) The first four stationary states of the harmonic oscillator. (b) Graph of $|\psi_{100}|^2$, with the classical distribution (dashed curve) superimposed.

Griffiths, QM